

Arithmetic Integer Additive Set-Indexers of Graph Operations

N K Sudev

Department of Mathematics

Vidya Academy of Science & Technology

Thrissur - 680501, India.

email: *sudevkn@gmail.com*

K A Germina

Department of Mathematics

School of Mathematical & Physical Sciences

Central University of Kerala

Kasaragod-671316, India.

email: *sgerminaka@gmail.com*

Abstract

An integer additive set-indexer is an injective function $f : V(G) \rightarrow 2^{\mathbb{N}_0}$ such that the induced function $g_f : E(G) \rightarrow 2^{\mathbb{N}_0}$ defined by $g_f(uv) = f(u) + f(v)$ is also injective. A graph G which admits an IASI is called an IASI graph. An arithmetic integer additive set-indexer is an integer additive set-indexer f , under which the set-labels of all elements of a given graph G are arithmetic progressions. In this paper, we discuss about admissibility of arithmetic integer additive set-indexers by certain graph operations and certain products of graphs.

Key words: Integer additive set-indexers, arithmetic integer additive set-indexers, isoarithmetic integer additive set-indexers, biarithmetic integer additive set-indexers.

AMS Subject Classification : 05C78

1 Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [11] and for more about graph labeling, we refer to [7]. Unless mentioned otherwise, all graphs considered here are simple, finite and have no isolated vertices. All sets mentioned in this paper are finite sets of non-negative integers. We denote the cardinality of a set A by $|A|$.

Definition 1.1. [8] An *integer additive set-indexer* (IASI, in short) is defined as an injective function $f : V(G) \rightarrow 2^{\mathbb{N}_0}$ such that the induced function $g_f : E(G) \rightarrow 2^{\mathbb{N}_0}$ defined by $g_f(uv) = f(u) + f(v)$ is also injective. A graph G which admits an IASI is called an IASI graph.

Definition 1.2. The cardinality of the labeling set of an element (vertex or edge) of a graph G is called the *set-indexing number* of that element.

In [9], the vertex set V of a graph G is defined to be *l -uniformly set-indexed*, if all the vertices of G have the set-indexing number l .

By the term, an arithmetically progressive set, (AP-set, in short), we mean a set whose elements are in arithmetic progression. We call the common difference of the set-label of an element of the given graph, the *deterministic index* of that element.

Proposition 1.3. Let f be a vertex-arithmetic IASI defined on G . If the set-labels of vertices of G are AP-sets with the same common difference d , then f is also an edge-arithmetic IASI of G .

Definition 1.4. [18] An *arithmetic integer additive set-indexer* is an integer additive set-indexer f , under which the set-labels of all elements of a given graph G are the sets whose elements are in arithmetic progressions. A graph that admits an arithmetic IASI is called an *arithmetic IASI graph*.

If all vertices of G are labeled by the set consisting of arithmetic progressions, but the set-labels are not arithmetic progressions, then the corresponding IASI may be called *semi-arithmetic IASI*.

Theorem 1.5. [18] A graph G admits an arithmetic IASI graph G if and only if for any two adjacent vertices in G , the deterministic index of one vertex is a positive integral multiple of the deterministic index of the other vertex and this positive integer is less than or equal to the cardinality of the set-label of the latter vertex.

Definition 1.6. [19] If all the set-labels of all elements of a graph G consist of arithmetic progressions with the same common difference d , then the corresponding IASI is called *isoarithmetic IASI*.

Definition 1.7. [20] Let f be an arithmetic IASI of a graph G . For two vertices v_i and v_j of G , let the common differences of $f(v_i)$ and $f(v_j)$ be d_i and d_j respectively. If either of d_i and d_j , for the adjacent vertices v_i and v_j , is a positive integral multiple of the other, then f is called a *biarithmetic IASI*. For $k \in \mathbb{N}_0$, if $d_i = k.d_j$ for all adjacent vertices v_i and v_j in G , then f is called a *biarithmetic IASI* of G .

If the value of k is unique for all pairs of adjacent vertices of a biarithmetic IASI graph G , then that biarithmetic IASI is called *identical biarithmetic IASI* and G is called an *identical biarithmetic IASI graph*.

Theorem 1.8. [18, 19, 20] A subgraph of an arithmetic (or isoarithmetic or biarithmetic) IASI graph is also an arithmetic IASI graph (or isoarithmetic or biarithmetic) IASI graph.

Theorem 1.9. [20] A graph G admits an identical biarithmetic IASI if and only if it is bipartite.

2 New Results

In this paper, we investigate the admissibility of arithmetic integer additive set-indexers by different operations and certain products of arithmetic IASI graphs.

2.1 Arithmetic IASIs of Graph Operations

The following result establishes the admissibility of the union of two arithmetic IASI graphs.

Proposition 2.1. *The union of two arithmetic IASI graphs admits an arithmetic IASI graph.*

Proof. let f_1 and f_2 be the arithmetic IASIs defined on G_1 and G_2 respectively. Define a function f on $G = G_1 \cup G_2$ by

$$f(v) = \begin{cases} f_1(v) & \text{if } v \in G_1 \\ f_2(v) & \text{if } v \in G_2. \end{cases}$$

Since both f_1 and f_2 are arithmetic IASIs, then f is also an arithmetic IASI on $G_1 \cup G_2$. \square

Proposition 2.2. *The union of two isoarithmetic IASI graphs admits an isoarithmetic IASI graph if and only if all the vertices in both G_1 and G_2 have the same deterministic index.*

Proof. Let f_1 and f_2 be the isoarithmetic IASIs defined on G_1 and G_2 respectively. Define a function f on $G = G_1 \cup G_2$ by

$$f(v) = \begin{cases} f_1(v) & \text{if } v \in G_1 \\ f_2(v) & \text{if } v \in G_2. \end{cases}$$

Assume that the vertices of both G_1 and G_2 have the same deterministic index. Then, all the vertices of $G_1 \cup G_2$ have the same deterministic index. Therefore, f is arithmetic IASI on $G_1 \cup G_2$.

Conversely, assume that $G_1 \cup G_2$ admits an isoarithmetic IASI, say f . Therefore, by Theorem 1.8, its subgraphs G_1 and G_2 also admit isoarithmetic IASIs which are the restrictions f_1 and f_2 of f to G_1 and G_2 respectively. \square

Now, recall the definition of the join of two graphs.

Definition 2.3. [11] Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs. Then, their *join* (or *sum*), denoted by $G_1 + G_2$, is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup E_{ij}$, where $E_{ij} = \{u_i v_j : u_i \in G_1, v_j \in G_2\}$.

Theorem 2.4. *The join of two arithmetic IASI graphs admits an arithmetic IASI if and only if the deterministic index of every vertex of one graph is an integral multiple or divisor of the deterministic index of every vertex of the other graph, where this integer is less than or equal to the set-indexing number of the vertex having smaller deterministic index.*

Proof. Let G_1 and G_2 be the given arithmetic IASI graphs. Let $E_{ij} = \{u_i v_j : u_i \in G_1, v_j \in G_2\}$ so that $G_1 + G_2 = G_1 \cup G_2 \cup \langle E_{ij} \rangle$.

Assume that $G_1 + G_2$ admits an arithmetic IASI, say f . Therefore, by Theorem 1.5, for all edges in $\langle E_{ij} \rangle$ also, the deterministic index of one end vertex is an integral multiple of the deterministic index of the other end vertex, where this integer is less than or equal to the cardinality of the set-label of the latter vertex. Since every vertex, say u_i , in G_1 is adjacent to every vertex, say v_j of G_2 (and vice versa), deterministic index of u_i is either a multiple or a divisor of the deterministic index of v_j , where this integer is less than or equal to the set-indexing number of the vertex having smaller deterministic index.

Conversely, assume, without loss of generality, that the deterministic index of every vertex of G_1 is a multiple or a divisor of the deterministic index of every vertex of G_2 such that this integer is less than or equal to the set-indexing number of the vertex having smaller deterministic index. Hence, for every edge in $G_1 + G_2$, the deterministic index of one end vertex is an integral multiple of the deterministic index of the other end vertex, since both G_1 and G_2 are arithmetic IASI graph. Then, by Theorem 1.5, $G_1 + G_2$ admits an arithmetic IASI. \square

The following results establish the admissibility of arithmetic and isoarithmetic IASIs by the join of two isoarithmetic IASI graphs.

Proposition 2.5. *The join of two isoarithmetic IASI graphs is an arithmetic IASI graph if and only if the deterministic index of the elements of one graph is an integral multiple of the deterministic index of the elements of the other, where this integer is less than or equal to the minimum among cardinalities of all set-labels of the former graph.*

Proof. Let G_1 and G_2 admit isoarithmetic IASIs f_1 and f_2 respectively and let $E_{ij} = \{u_i u_j : u_i \in G_1, u_j \in G_2\}$ be such that $G_1 + G_2 = G_1 \cup G_2 \cup \langle E_{ij} \rangle$. Note that all the elements of G_1 have the same deterministic index, say d_1 and all the elements of G_2 also have the same deterministic index, say d_2 . Let that $d_1 \neq d_2$.

Now, assume that the sum of two isoarithmetic IASI graphs is an arithmetic IASI graph. Then, by Theorem 1.5, for all edges in E_{ij} , the deterministic index of one end vertex is an integral multiple of the deterministic index of the other, with this integer is less than or equal to the cardinality of the set-label of the former vertex. Without loss of generality, let $d'_j = k d_i$, where d_i is the deterministic index of the vertex v_i in G_1 and d'_j is the deterministic index of the vertex u_j in G_2 and k is a positive integer $|f_1(v_i)|$. Therefore, the deterministic index of the elements of G_2 is an integral multiple of the deterministic index of the elements of G_1 , where this integer is less than or equal to the minimum among the cardinalities of all set-labels of the G_1 .

Conversely, assume, without loss of generality, that the deterministic index of the elements of G_2 is an integral multiple of the deterministic index of the elements of G_1 , where this integer is less than or equal to the minimum among the cardinalities of all set-labels of G_1 . Therefore, by Theorem 1.5, $G_1 + G_2$ admits an arithmetic IASI. \square

Proposition 2.6. *The join of two isoarithmetric IASI graphs admits an isoarithmetric IASI if and only if all the vertices in both G_1 and G_2 have the same deterministic index.*

Proof. First assume that $G_1 + G_2$ admits an isoarithmetric IASI, say f . Then, the deterministic index of all vertices of $G_1 + G_2$ is the same, say d . Since, G_1 and G_2 are subgraphs of $G_1 + G_2$, by Theorem 1.8, G_1 and G_2 admits the induced isoarithmetric IASI of f . All the vertices in both G_1 and G_2 have the same deterministic index.

Conversely, the vertices in both G_1 and G_2 have the same deterministic index. Let $E_{ij} = \{u_i v_j : u_i \in G_1, v_j \in G_2\}$. Then, for any edge in $\langle E_{ij} \rangle$ must the deterministic index its end vertices are the same as that of both G_1 and G_2 . Hence, by Theorem 1.5, $G_1 + G_2$ admits an isoarithmetric IASI. \square

Proposition 2.7. *The join of two isoarithmetric IASI graphs is an arithmetic IASI graph if and only if the deterministic index of the elements of one graph is an integral multiple of the deterministic index of the elements of the other.*

Proposition 2.8. *Let G_1 and G_2 be two graphs which admit identical biarithmetric IASI. Then, $G_1 + G_2$ does not admit an identical biarithmetric IASI.*

Proof. If possible, let $G_1 + G_2$ admits an identical biarithmetric IASI. Since G_1 and G_2 are identical biarithmetric IASI graphs, for every pair of adjacent vertices in them, the deterministic index of one is a positive integral multiple of the deterministic index of the other and this positive integer is unique for all such pair vertices in G_1 and G_2 . Let v_i be a vertex of G_1 with deterministic index d_i and let u_j and u_l be two adjacent vertices in G_2 with deterministic indices d'_j and d'_l respectively. By Theorem 1.5, $d'_l = k \cdot d'_j$ for some positive integer k .

Now, v_i is adjacent to both u_j and u_l in $G_1 + G_2$. Then, by Theorem 1.5, we have $d_i = k \cdot d'_j$, $d'_l = k \cdot d'_j$ and $d_i = k \cdot d'_l$, all of which cannot hold simultaneously. Therefore, $G_1 + G_2$ does not admit an identical biarithmetric IASI. Hence, $G_1 + G_2$ does not admit an identical biarithmetric IASI. \square

2.2 Arithmetic IASIs of Graph Products

We discuss the admissibility of arithmetic IASI by certain graph products. First, recall the definition of the cartesian product of two graphs.

Definition 2.9. [11] Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs. Then, the *cartesian product* of G_1 and G_2 , denoted by $G_1 \times G_2$, is the graph with vertex set $V_1 \times V_2$ defined as follows. Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ be two points in $V_1 \times V_2$. Then, u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1 \text{ and } u_2 \text{ is adjacent to } v_2]$ or $[u_2 = v_2 \text{ and } u_1 \text{ is adjacent to } v_1]$. If $|V_i| = p_i$ and $E_i = q_i$ for $i = 1, 2$, then $|V(G_1 \times G_2)| = p_1 p_2$ and $|E(G_1 \times G_2)| = p_1 q_2 + p_2 q_1$.

The cartesian product $G_1 \times G_2$ may be viewed as follows. Make p_2 copies of G_1 . Denote these copies by G_{1i} , which corresponds to the vertex v_i of G_2 . Now, join the corresponding vertices of two copies G_{1i} and G_{1j} if the corresponding vertices v_i and v_j are adjacent in G_2 . Thus, we view the product $G_1 \times G_2$ as a union of p_2 copies of

G_1 and a finite number of edges connecting two copies G_{1i} and G_{1j} of G_1 according to the adjacency of the corresponding vertices v_i and v_j in G_2 , where $1 \leq i \neq j \leq p_2$.

The following theorem establishes the admissibility of arithmetic IASI by the cartesian product of two arithmetic IASI graphs.

Theorem 2.10. *The cartesian product of two arithmetic IASI graphs G_1 and G_2 admits an arithmetic IASI if and only if, for any pair of corresponding vertices of the copies of G_1 (or G_2) which are adjacent in $G_1 \times G_2$, the deterministic index of one vertex is an integral multiple (or a divisor) of the deterministic index of the other vertex, where this integer is less than or equal to the set-indexing number of the vertex having smaller deterministic index.*

Proof. Let $U = \{u_1, u_2, u_3, \dots, u_n\}$ be the vertex set of G_1 and $V = \{v_1, v_2, v_3, \dots, v_m\}$ be the vertex set of G_2 . Let G_{1j} ; $1 \leq j \leq m$, be the m copies of G_1 in $G_1 \times G_2$. Therefore, $G_{1j} = \langle U_j \rangle$ where $U_j = \{u_{ij} : 1 \leq i \leq |E(G_1)|, 1 \leq j \leq |E(G_2)|\}$. Now, for all values of j , the graphs induced by the set of vertices $\{u_{ij} : 1 \leq i \leq |E(G_1)|\}$ are graphs isomorphic of G_1 and similarly for all values of i , the graphs induced by the set of vertices $\{u_{ij} : 1 \leq j \leq |E(G_2)|\}$ are the graphs isomorphic of G_2 . Without loss of generality, let $\langle U_1 \rangle = G_1$ and $\langle V_1 \rangle = G_2$, where $V_1 = \{u_{i1} : 1 \leq i \leq |E(G_2)|\}$. Also, the corresponding vertices of G_{1r} and G_{1s} are adjacent in $G_1 \times G_2$ if the vertices v_r and v_s adjacent in G_2 .

Now, let f_1 and f_2 be the arithmetic IASIs of G_1 and G_2 respectively. Since G_1 is an arithmetic IASI graph, for two adjacent vertices u_r and u_s in G_1 , we have $d_s = k_l \cdot d_r$, where $k_l \leq |f_1(u_r)|$, is a positive integer with $1 \leq l \leq |E(G_1)|$. Similarly, Since G_2 is an arithmetic IASI graph, for two adjacent vertices v_r and v_s in G_2 , we have $d'_s = k'_l \cdot d'_r$, where $k'_l \leq |f_2(v_r)|$, is a positive integer with $1 \leq l \leq |E(G_2)|$, where d_i is the deterministic index of the vertex u_i in G_1 and d'_j is the deterministic index of the vertex v_j in G_2 . Label the vertices of $\langle U_1 \rangle$ by the same set-labels of G_1 itself and label the vertices of the copies $\langle U_r \rangle$ and $\langle U_s \rangle$ in such a way that the deterministic indices of all vertices in U_s are integral multiple of the deterministic indices of the corresponding vertices of U_r by a unique positive integer k'_l , k'_l being the same positive integer given by $k'_l = \frac{d'_s}{d'_r}$ where d'_r and d'_s are the deterministic indices of the vertices v_r and v_s in G_2 corresponding to the copies U_r and U_s respectively. Then, for every pair of adjacent vertices in $G_1 \times G_2$, the deterministic index of one vertex is an integral multiple of the deterministic index of the other. Hence, by Theorem 1.5, $G_1 \times G_2$ is an arithmetic IASI graph.

Conversely, assume that $G_1 \times G_2$ admits an arithmetic IASI. Now we can take the same set-labels of the copy U_1 as the set-labels of the vertices of G_1 and the same set-labels of the graph $\langle V_1 \rangle$, defined above, as the set-labels of the graph G_2 . Since the set-labels of all elements of $G_1 \times G_2$ are AP-sets, these set-labelings of G_1 and G_2 are arithmetic IASIs. Therefore G_1 and G_2 are arithmetic IASI graphs. \square

Invoking Proposition 2.10, we now establish the following results.

Corollary 2.11. *The cartesian product of two isoarithmic IASI graphs admits an isoarithmic IASI if and only if all vertices in both G_1 and G_2 hve the same deterministic index.*

Proof. The proof is immediate from Theorem 2.10 by taking $k_l = 1$ and $k'_l = 1$. \square

Now, we investigate the admissibility of an arithmetic IASI by the cartesian product of two identical biarithmetic IASI graphs.

Invoking Theorem 1.9 and Theorem 2.10 we establish the following result.

Proposition 2.12. *The cartesian product of two identical biarithmetic IASI graphs admits an identical biarithmetic IASI.*

Proof. Since G_1 and G_2 are identical biarithmetic IASI graphs, by Theorem 1.9, both G_1 and G_2 are bipartite. Therefore, since the cartesian product of two bipartite graphs is also a bipartite graph, $G_1 \times G_2$ is bipartite. Hence, $G_1 \times G_2$ admits an identical biarithmetic IASI. The labeling of the vertices in $G_1 \times G_2$ follows from Theorem 2.10 by taking $k_l = k'_l = k$, a unique positive integer. \square

Next, we proceed to verify the admissibility of arithmetic IASI by the corona of two graphs. Now, recall the definition of corona of two graphs.

Definition 2.13. [11] By the term *corona* of two graphs G_1 and G_2 , denoted by $G_1 \circ G_2$, is the graph obtained taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i -th point of G_1 to every point in the i -th copy of G_2 .

The number of vertices and edges in $G_1 \circ G_2$ are $p_1(1 + p_2)$ and $q_1 + p_1q_2 + p_1p_2$ respectively, where p_i and q_i are the number of vertices and edges of the graph $G_i, i = 1, 2$.

Theorem 2.14. *Let G_1 and G_2 are an arithmetic IASI graphs. Then, the corona $G_1 \circ G_2$ admits an arithmetic IASI if and only if the deterministic index of every vertex of one graph is an integral multiple or a divisor of the deterministic index of every vertex of the other, where this integer is less than or equal to the set-indexing number of the vertex having smaller deterministic index..*

Proof. Let $U = \{u_1, u_2, u_3, \dots, u_n\}$ be the vertex set of G_1 and $V = \{v_1, v_2, v_3, \dots, v_m\}$ be the vertex set of G_2 . Let $G_{2i}; 1 \leq i \leq n$, be the n copies of G_2 in $G_1 \circ G_2$. Therefore, $G_{2i} = \langle V_i \rangle$ where $V_i = \{v_{ij} : 1 \leq i \leq |E(G_1)| \text{ and } 1 \leq j \leq |E(G_2)|\}$. Now, for any value of $i = r$, the graphs induced by the set of vertices $\{v_{rj} : 1 \leq j \leq |E(G_2)|\}$ is a graph isomorphic of G_2 . Without loss of generality, let $\langle V_1 \rangle = G_2$. Also, all the vertices of $\langle V_i \rangle$ are adjacent to the vertex u_i of G_1 in $G_1 \circ G_2$.

Now, assume that G_1 and G_2 admit arithmetic IASIs f_1 and f_2 respectively. Since G_1 is an arithmetic IASI graph, for two adjacent vertices u_r and u_s in G_1 , we have $d_s = k_l \cdot d_r$, where $k_l \leq |f_1(u_r)|$ is a positive integer with $1 \leq l \leq |E(G_1)|$. Similarly, since G_2 is an arithmetic IASI graph, for two adjacent vertices v_r and v_s in G_2 , we have $d'_s = k'_l \cdot d'_r$, where $k'_l \leq |f_2(v_r)|$ is a positive integer with $1 \leq l \leq |E(G_2)|$, where d_i is the deterministic index of the vertex u_i in G_1 and d'_j is the deterministic index of the vertex v_j in G_2 . Label the vertices of $\langle V_1 \rangle$ by the same set-labels of G_2 itself and label the vertices of the copies $\langle V_r \rangle; 1 < r \leq n$, by distinct sets in such a way that the deterministic indices of the corresponding vertices in V_1 and V_r have the

same deterministic indices. Then, for every pair of adjacent vertices in $G_1 \circ G_2$, the deterministic index of one vertex is an integral multiple of the deterministic index of the other. Hence, by Theorem 1.5, $G_1 \circ G_2$ is an arithmetic IASI graph.

Conversely, assume that $G_1 \circ G_2$ admits an arithmetic IASI. Since G_1 is a subgraph of $G_1 \circ G_2$, by Theorem 1.8, G_1 admits an arithmetic IASI. Also, we can take the same set-labels of the copy V_1 as the set-labels of the vertices of G_2 and the same set-labels of the component graph $\langle V_1 \rangle$ of $G_1 \circ G_2$, defined above, as the set-labels of the graph G_2 . Since the set-labels of all elements of $G_1 \times G_2$ are AP-sets, these set-labelings of G_1 and G_2 are arithmetic IASIs. Therefore G_1 and G_2 are arithmetic IASI graphs. \square

Invoking Proposition 2.10, we now establish the following results.

Proposition 2.15. *The corona of two isoarithmic IASI graphs admits an isoarithmic IASI if and only if all vertices in both G_1 and G_2 have the same deterministic index.*

Proof. The proof is immediate from Theorem 2.14. \square

Proposition 2.16. *The corona of two identical biarithmic IASI graphs does not admit an identical biarithmic IASI.*

Proof. Since the corona of two bipartite graphs is not a bipartite graph, by theorem 1.9, $G_1 \circ G_2$ does not admit an identical biarithmic IASI. \square

2.3 Arithmetic IASIs of Graph Complements

In this section, we discuss the admissibility of arithmetic IASI by complements of given arithmetic IASI graphs. Note that the vertices of a graph G and its complements have the same set-labels and hence the same deterministic indices.

Theorem 2.17. *The complement of an arithmetic IASI graph G admits an arithmetic IASI if and only if the deterministic index of any vertex of G is an integral multiple or divisor of the deterministic index of every other vertex of G .*

Proof. First assume that the deterministic index of any vertex of G is an integral multiple or divisor of the deterministic index of every other vertex of G . Hence, for any pair of adjacent vertices in \bar{G} also, the deterministic index of one vertex is an integral multiple of the deterministic index of the other. Hence by Theorem 1.5, \bar{G} admits an arithmetic IASI.

Conversely, assume that \bar{G} admits an arithmetic IASI. Since every pair of vertices in $V(G)$ are either adjacent in G or in its complement \bar{G} and both G and \bar{G} are arithmetic IASI graphs, for every pair of vertices the deterministic index of one vertex must be a multiple of the deterministic index of the other. \square

Proposition 2.18. *The complement of an isoarithmic IASI graph is also an isoarithmic IASI graph.*

Proof. Let G be an isoarithmic IASI graph. Then, the deterministic index of all vertices of G (and \bar{G}) are the same. Therefore, \bar{G} admits an isoarithmic IASI. \square

Proposition 2.19. *The complement of an identical biarithmic IASI never admits an identical biarithmic IASI.*

Proof. The complement of a bipartite graph G is not a bipartite graph. Hence, by Theorem 1.9, \bar{G} does not admit an identical biarithmic IASI. \square

3 Conclusion

In this paper, we have discussed some characteristics of certain graphs operations and products which admit arithmetic IASIs. We have not addressed certain Problems in this area which are still open.

The following are some of the open problems we have identified in this area.

Problem 1. Examine the necessary and sufficient conditions for the existence of non-identical biarithmic IASIs for given graphs.

Problem 2. Discuss the necessary and sufficient conditions for the existence of arithmetic IASIs for some other products, such as lexicographic product, tensor product, strong product, rooted product etc., of arithmetic IASI graphs.

Problem 3. Discuss the necessary and sufficient conditions for the existence of arithmetic IASIs of all types for certain powers of arithmetic IASI graphs.

Problem 4. Discuss the necessary and sufficient conditions for the existence of arithmetic IASIs for different graph classes having arithmetic IASIs.

Problem 5. Characterise certain graphs and graph classes in accordance with their admissibility of identical and non-identical biarithmic IASIs.

The IASIs under which the vertices of a given graph are labeled by different standard sequences of non negative integers, are also worth studying. The problems of establishing the necessary and sufficient conditions for various graphs and graph classes to have certain IASIs still remain unsettled.

References

- [1] B D Acharya, (1990). *Arithmetic Graphs*, J. Graph Theory, **14**(3), 275-299.
- [2] B D Acharya, K A Germina and T M K Anandavally, *Some New Perspective on Arithmetic Graphs In Labeling of Discrete Structures and Applications*, (Eds.: B D Acharya, S Arumugam and A Rosa), Narosa Publishing House, New Delhi, (2008), 41-46.
- [3] J A Bondy and U S R Murty, (1976). **Graph Theory with Applications**, North-Holland, Amsterdam.

- [4] G Chartrand and P Zhang, (2005). **Introduction to Graph Theory**, McGraw-Hill Inc.
- [5] N Deo, (1974). **Graph Theory with Applications to Engineering and Computer Science**, Prentice-Hall.
- [6] R Frucht and F Harary (1970). *On the Corona of Two Graphs*, Aequationes Math., **4**(3), 322-325.
- [7] J A Gallian, (2011). *A Dynamic Survey of Graph Labelling*, The Electronic Journal of Combinatorics (DS 16).
- [8] K A Germina and T M K Anandavally, (2012). *Integer Additive Set-Indexers of a Graph: Sum Square Graphs*, Journal of Combinatorics, Information and System Sciences, **37**(2-4), 345-358.
- [9] K A Germina, N K Sudev, (2013). *On Weakly Uniform Integer Additive Set-Indexers of Graphs*, Int. Math. Forum., **8**(37), 1827-1834.
- [10] R Hammack, W Imrich and S Klavzar (2011). *Handbook of Product graphs*, CRC Press.
- [11] F Harary, (1969). **Graph Theory**, Addison-Wesley Publishing Company Inc.
- [12] S M Hegde, (1989). *Numbered Graphs and Their Applications*, PhD Thesis, Delhi University.
- [13] W Imrich, S Klavzar, (2000). **Product Graphs: Structure and Recognition**, Wiley.
- [14] W Imrich, S Klavzar and D F Rall, (2008). **Topics in Graph Theory: Graphs and Their Cartesian Products**, A K Peters.
- [15] K D Joshi, **Applied Discrete Structures**, New Age International, (2003).
- [16] M B Nathanson (1996). **Additive Number Theory, Inverse Problems and Geometry of Sumsets**, Springer, New York.
- [17] N K Sudev and K A Germina, (2014). *A Note on Integer Additive Set-Indexers of Graphs*, Int. J. Math. Sci. & Engg. Applications, **8**(2), 11-22.
- [18] N K Sudev and K A Germina, *On Arithmetic Integer Additive Set-Indexers of Graphs*, submitted.
- [19] N K Sudev and K A Germina, *On Isoarithmetic Integer Additive Set-Indexers of Graphs*, submitted.
- [20] N K Sudev and K A Germina, *Biarithmetic Integer Additive Set-Indexers of Graphs*, submitted.
- [21] R J Trudeau, (1993). **Introduction to Graph Theory**, Dover Pub., New York.
- [22] D B West, (2001). **Introduction to Graph Theory**, Pearson Education Inc.